Designing Pricing Incentive Mechanism for Proactive Demand Response in Smart Grid

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***Abstract*—Demand side management will be a key component of future smart grid that can help reduce peak load and adapt elastic demand to fluctuating generations. In this paper, we consider customers that operate different appliances and propose a demand response approach based on utility maximization. Each appliance provides a certain benefit depending on the pattern or volume of power it consumes. Each customer wishes to optimally schedule its power consumption so as to maximize its individual net benefit subject to various consumption and power flow constraints. We show that there exist time-varying prices that can align individual optimality with social optimality, i.e., under such prices, when the customers selfishly optimize their own benefits, they automatically also maximize the social welfare. The utility company can thus use dynamic pricing to coordinate demand responses to the benefit of the overall system. We propose a distributed algorithm for the utility company and the customers to jointly compute this optimal prices and demand schedules. Finally, we present simulation results that illustrate several interesting properties of the proposed scheme.**

I. Introduction

The ever increasing power consumption of urban area electric power distribution is expected to contribute to 40% of global electricity energy consumption in the five upcoming years [1]. Furthermore, elastic appliances (e.g. energy storage systems (ESS), electric vehicles (EVs)) are deployed closer to the customers. However, these appliances are connected to a core network via capacity-limited transmission links which make it difficult to meet customers' requirements in terms of satisfaction, especially during peak hours. To deal with this problem, demand response (DR) in electric power distribution has recently been proposed as a promising solution [2].

The idea of DR is allowing customers to participate in the electricity markets by coordinating interaction between utility company [3]. Thus, the elastic appliances can serve most of the request locally without using back haul. However, for a successful deployment of proactive load scheduling, the utility company requires cooperation of customers to be able to schedule their appliances. To this end, incentive mechanism must be deployed by utility company to incite customers to coordinate their multi-class appliances. The utility company offers customers DR service that allows customers to improve their satisfaction and in turn the price is set by utility company depending on the amount of power consumption requested by each customer.

Recently, several DR programes have appeared from different aspects, such as optimal DR based on utility maximization in power networks [4], multi-class appliances scheduling DR in residential area [5], distributed DR with energy storage in smart grid [6] and so on. More relevantly, there exist some works focusing on pricing incentive scheme. In such schemes, the utility company defines pricing scheme to motivate users to proactively schedule power transmission rate of their appliances to meet their satisfaction. In [7], the authors proposed a time-of-use pricing scheme for residential load scheduling. In [6], based on game theory and proximal decomposition, the authors offered two distributed incentive algorithms executed by users to encourage them to actively participate in shifting their energy usage from the peak-load periods to the low-demand periods. In [8], the authors formulated the pricing incentive problem as a Stackelberg game, in which an energy provider acts as a leader and customers act as followers. The dynamic strategies from the energy provider and the users are given, and a distributed algorithm has been proposed where the link between the leader and followers is the price signal.

Despite being interesting, all these works focused on DR incentive mechanism in electric power distribution and ignored modeling preference for customers and operating properties of multi-class appliances. The main contribution of this work are to propose a new pricing incentive mechanism between utility company and multiple customers considering their preference and appliances properties. We formulate the DR problem as a Stackelberg game in which the utility company is a leader and customers are the followers. The utility company defines the price that maximizes weighted summation of its revenue and peak-to-average ratio (PAR). On the other hand, we analytically model customers' preference and and their appliance patterns in form of selected utility function based on concepts from microeconomics. The competition between customers is formulated as a non-cooperative sub-game in which each customer aims to maximize the utility in terms of power transmission rate. Both the existence and uniqueness of the equilibrium is proved. The equilibrium represents a state in which none of these customers can improve its benefit

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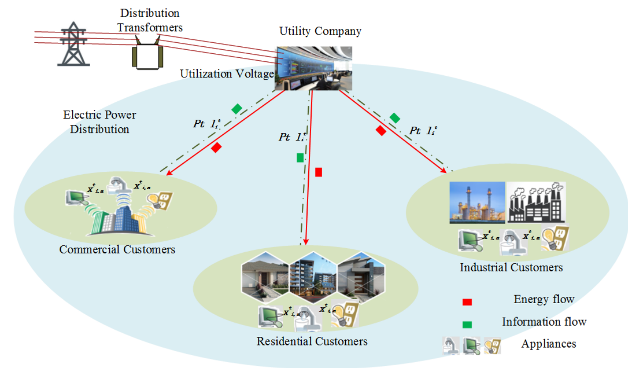

Fig. 1. Electric power distribution is the final stage in the delivery of electric power; it carries electricity from the transmission system to individual consumers. Distribution transformers lower voltage to the utilization voltage. Its distribution network consists of single utility company; a cluster of customers; and numerous appliances. The flow of electricity within the system is maintained and controlled by utility company which can adopt pricing incentive DR based on customers requirements.

by requesting a different amount of power transmission rate given the requested amount by the other customers fixed. We derive the close-form expressions of power scheduling rate customers must request at the equilibrium. Then, given the request power transmission rate of customers, an optimization problem is formulated at the utility company to determine the optimal price that should be charged to customers to maximize its revenue and minimize PAR. Simulation results show that customers can get much more economic saving and utility company get higher revenue and lower PAR compared to the case in which the prices are selected arbitrarily. Moreover, at the equilibrium, we demonstrate the performance with different tradeoff criteria and find that when the criteria are assigned to 0.9, utility company and customers can jointly achieve optimal solution.

The rest of this paper is organized as follows. In Section II, we provide the system model and mathematical models for appliances, costomers preference and utility function model and utility company utility function model. In Section III, we formulate two-stage Stackelberg based DR scheduling algorithm in distributed manner. In Section VI, we provide numerical results and finally we conclude in Section V.

II. System Model

We consider an urban electric power distribution system which consists of one utility company, various customers and numerous appliances. Energy management controllers (EMCs), which are embedded in a smart meter at customers' side, reasonably modify the behavior of power consumption by using the electricity price and customers preferences, as shown in Fig. 1. When the real-time electricity price p_t delivered by utility company to the communication network is received by a smart meter, the EMC will control the power consumption across customers' home. Denote $\mathcal{T}$ as the set of time slots $t \in \mathcal{T}$, $T = |\mathcal{T}|$. There is set of $\mathcal{N}$ customers that are willing to participate in the pricing incentive DR procedure in order to enhance the satisfaction of themselves, $i \in \mathcal{N}$, $N = |\mathcal{N}|$. Each customer operates set of $\mathcal{A}$ appliances, $a \in \mathcal{A}$. Let $x_{i,a}^t$ be the power consumption of customer i for time slot t by appliance a, $a \in \mathcal{A}$. $l_i^t = \sum_a^{a\in\mathcal{A}} x_{i,a}^t$ denotes power rate transmitted between utility company and customer i in time slot t. $x_{i,a}^t$ is constraint by operational properties of appliance $a \in \mathcal{A}$, which models in following subsection.

A. Operational Properties of Appliances Model

The total set $\mathcal{A}$ of appliances is classified into inelastic appliances, elastic appliances with memoryless property and elastic appliances with memory property, as $\{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3\}$ [5].

- $\mathcal{A}_1$: Appliances can be intermittently turned on and off with any performance degradation.
- $\mathcal{A}_2$: Appliances can be flexibly adjusted to satisfy user's perference with controllable energy density, e.g. lights bulbs with controllable brightness.
- $\mathcal{A}_3$: Appliances can be flexibly adjusted in two directions and depends not only on power rate, but on previous memory, e.g. ESS, EVs.

Appliances $a \in \{\mathcal{A}_1, \mathcal{A}_2\}$ has the maximum and minimum power consumption rate $x_{i,a}^{t,\max}$ and $x_{i,a}^{t,\min}$, respectively. For appliances $a \in \mathcal{A}_1$ can be intermittently turned off in time slot t, $x_{i,a}^{t,\min} = 0$, which is defined as:

$$\begin{cases} 0 \le x_{i,a}^t \le x_{i,a}^{t,\max}, & \forall a \in \mathcal{A}_1, i \in \mathcal{N}, t \in \mathcal{T}, \\ x_{i,a}^{t,\min} \le x_{i,a}^t \le x_{i,a}^{t,\max}, & \forall a \in \mathcal{A}_2, i \in \mathcal{N}, t \in \mathcal{T}. \end{cases} \tag{1}$$

Appliances $a \in \mathcal{A}_3$ usually have an upper bound on charging rate (when $x_{i,a}^t > 0$), denoted by $x_{i,a}^{t,\max}$, and an upper bound on discharging rate (when $x_{i,a}^t < 0$), denoted by $x_{i,a}^{t,\min}$. Let $B_{i,a}^{\text{cap}}$ denote memory-based storage capacity and $b_{i,a}^0$ denote initial memory level. Thus, there are following constraints on appliances $a \in \mathcal{A}_3$.

$$\begin{cases} x_{i,a}^{\min} \le x_{i,a}^t \le x_{i,a}^{\max}, & \forall a \in \mathcal{A}_3, i \in \mathcal{N}, t \in \mathcal{T}, \\ 0 \le B_{i,a}^t \le B_{i,a}^{\text{cap}}, & \forall a \in \mathcal{A}_3, i \in \mathcal{N}, t \in \mathcal{T}, \end{cases} \tag{2}$$

where $B_{i,a}^t$ denotes the dynamics of memory-based storage level and power leakage is negligible.

$$B_{i,a}^t = \sum_{t=1}^{t} x_{i,a}^t + b_{i,a}^0, \forall a \in \mathcal{A}_3, i \in \mathcal{N}, t \in \mathcal{T}. \tag{3}$$

Considering the high expense of $a \in \mathcal{A}_3$, the economic damage is modeled by a function $C_i(\boldsymbol{x}_{i,a})$ $(a \in \mathcal{A}_3)$ that depends on the vector of charged/discharged rate $\boldsymbol{x}_{i,a} := (x_{i,a}^t, t \in \mathcal{T}, a \in \mathcal{A}_3)$. This captures extra damages on elastic appliances with memory property by charging and discharging operations [4],

$$\begin{aligned} &C_i(\boldsymbol{x}_{i,a}) = \\ &B_{\alpha_i} \sum_{t}^{T} (x_{i,a}^t)^2 + B_{\gamma_i} \sum_{t=1}^{T} (\min(B_{i,a}^t - \delta_i B_{i,a}^{cap}, 0))^2, \\ &\forall a \in \mathcal{A}_3, i \in \mathcal{N}, t \in \mathcal{T}, \end{aligned} \tag{4}$$

where $B_{\alpha_i}, B_{\gamma_i}, \delta_i > 0$ are constant weight factors [9]. By appropriately controlling weight factors, users can tradeoff economic achievement and cost corresponding to fast charging and deep discharging.

B. Customers' Preference and Utility Function Model

In electric power distribution, customers perform as independent entities whose demands vary from different conditions, such as type of customers, shape of supply-demand and sensitivity to price. The response of heterogeneous customers to the above conditions is modeled by adopting the concept of *satisfaction function* from microeconomics [10]. The corresponding satisfaction function $s_i^t(l_i^t, \beta_i^t)$ quantitatively models customers' satisfaction level of actual power transmission with utility company, where β_i^t characterizes the differences in customers types. We assume that the satisfaction functions to fulfill the following properties [11]:

1) Property I: Customers are always interested to consume more power if possible until they reach their maximum consumption level. Mathematically, $s_i^t(l_i^t, \beta_i^t)$ is non-decreasing and also the marginal benefit are always larger than zero.

$$\frac{\partial s_i^t(l_i, \beta_i^t)}{\partial l_i} \geq 0, \forall i \in \mathcal{N}, t \in \mathcal{T}. \tag{5}$$

2) Property II: The level of customers' satisfaction gradually gets saturated. In other words, the marginal benefit of customers are non-increasing and $s_i^t(l_i^t, \beta_i^t)$ are concave.

$$\frac{\partial^2 s_i^t(l_i, \beta_i^t)}{\partial l_i^2} \leq 0, \forall i \in \mathcal{N}, t \in \mathcal{T}. \tag{6}$$

3) Property III: $s_i^t(l_i^t, \beta_i^t)$ can be ranked for the same power consumption, in order to varify the difference from customers types. As for the same consumption level l_i^t, a smaller β_i^t indicates a larger satisfaction.

$$\frac{\partial s_i^t(l_i^t, \beta_i^t)}{\partial \beta_i^t} < 0, \forall i \in \mathcal{N}, t \in \mathcal{T}. \tag{7}$$

4) Property IV: There are no benefit for no power consumption.

$$s_i^t(0, \beta_i^t) = 0, \forall i \in \mathcal{N}, t \in \mathcal{T}. \tag{8}$$

Given p_t, customers are willing to maximize the sum of overall welfare. The utility function of customers are defined as the sum of satisfaction and economic cost, denoted by $U_i(l_i^t, x_{i,a}^t)$ [12]:

$$\begin{aligned} U_i^t(l_i^t, x_{i,a}^t, p_t) = -p_t l_i^t - C_i(x_{i,a}^t) + s_i^t(l_i^t, \beta_i^t), \\ \forall a \in \mathcal{A}, i \in \mathcal{N}, t \in \mathcal{T}. \end{aligned} \tag{9}$$

Through EMCs, customers have the ability to decide power consumption rate of appliances across home to maximize its utility for announced p_t in each time slot t. The utility maximum optimization problem can be characterized as the solution to the following problem:

$$\begin{aligned} &\max_{\substack{\{l_i^t, x_{i,a}^t, p_t\}, \\ l_{i,\min}^t \leq l_i^t \leq l_{i,\max}^t}} && \sum_{i \in \mathcal{N}} U_i^t(l_i^t, x_{i,a}^t, p_t) \\ &\text{subject to} && (1), (2), (3). \end{aligned} \tag{10}$$

The utility function (9) is a concave maximization problem and the feasible set is convex. Hence an optimal point can be solved by *convex programming* techniques, such as interior point method (IPM) in a central fashion. However, there arises a tough problem that the exact utility function of customers should be known by utility company caused by the central manner solution. Since the optimization problem can be solved by local EMCs in distributed fashion, the private parameter β_i^t is protected by distributed algorithm design that described in section III.

C. Utility Function Model of Utility Company

Distribution company serves as an intermediary that participates in wholesale markets, wishes to maximize its profits, as well as fulfill obligation to serve public and satisfy costumers. So, utility company is not to maximize its profit, but rather induce customers to schedule power rate of appliances cooperatively by pricing incentive mechanism, and in return maximize social welfare. Hence the utility function of utility company is defined as the weighted summation of economical profits and squared consumption deviations to average (also called PAR) [13]:

$$\begin{aligned} S(p_t, l_i^t) = &\sum_{i \in \mathcal{N}} \left(w_u \sum_{t \in \mathcal{T}} (p_t - m_t) l_i^t - w_c (l_i^t - \bar{l}_i)^2 \right), \\ &\forall i \in \mathcal{N}, t \in \mathcal{T}, \end{aligned} \tag{11}$$

where m_t is marginal price of electricity, $\bar{l}_i = \frac{1}{T} \sum_{t \in \mathcal{T}} l_i^t$ ($\forall i \in \mathcal{N}$) is the average electricity demand during the day. $w_u \geq 0$ and $w_c \geq 0$ are weight factors that measure total utility budget and customers' daily consumption deviation, respectively. Therefore, the optimization problem of utility company is formulated as:

$$\begin{aligned} &\max_{\substack{\{p_t, l_i^t\} \\ Q_t^{\min} \leq Q_t \leq Q_t^{\max}}} && S(p_t, l_i^t) \\ &\text{subject to} && p_t^{\min} \leq p_t \leq p_t^{\max}, \forall t \in \mathcal{T}, \end{aligned} \tag{12}$$

where $Q_t = \sum_{i \in \mathcal{N}} l_i^t$ indicates overall power consumption of customers in time slot t in the electric power distribution, $Q_t^{\min}$ and $Q_t^{\max}$ are threshold for Q_t. In this problem, if $w_u > 0$, $w_c = 0$, the problem becomes a maximization problem of total utility budget. On the other hand, if $w_u = 0$ and $w_c > 0$, the problem becomes a minimization problem with PAR. By appropriately deciding weight factors, utility company can adjust weights parameters to tradeoff economic achievement and PAR. Note that the above problem is convex problem, which can be solved by *convex programming* techniques.

III. Two-stage Stackelberg Pring Incentive DR Mechanism Design

In this section, we formulate two stage Stackelberg game model [14] to capture the interactions between utility company and customers. The first stage of this game is that utility company determines price p_t, the second stage is that customers decide how much power to consume of appliances $x_{i,a}^t$. It is natural to assume that the utility company is the first mover and customers are followers that make their decision according to the prices. To obtain the Stackelberg equilibrium of the game, we can use the backward induction [14]. In particular, we first consider the energy consumption determined by customers for given price. By knowing customers' best responsive demands, utility company decides its optimal pricing scheme.

A. Optimal Response of Heterogeneous Customers

Since not all customers response to price in the same way, customers' preference model (5)-(8) in Section II can be extended to multi-type customers scenarios. In this paper, we define customers' satisfaction function as follows [15], [16]:

$$\begin{aligned} s_i^t(l_i^t, \beta_i^t) &= c_i^t l_{i,\max}^t f_i^t(l_i^t, \beta_i^t), i \in \mathcal{N}, t \in \mathcal{T}, \\ f_i^t(l_i^t, \beta_i^t) &= (\omega_i^t)^{\beta_i^t}, 0 < \beta_i^t \leq 1, i \in \mathcal{N}, t \in \mathcal{T}, \end{aligned} \tag{13}$$

where $\omega_i^t = l_i^t / l_{i,\max}^t$ denotes the ratio between actual power transmission and maximal possible demand transmission with utility company in normalized form. Let c_i^t denote per unit valuation of power transmission for customer i and it is independent from time. c_i^t is varied from different customer types, e.g. industrial-type customers valuation can be much great than that of residential-type customers.

In order to analyze the Stackelberg equilibrium, we use backward induction and first consider the second stage of the game, i.e, given p_t, customers maximize their welfare function (10) by choosing the power transmission rate l_i^t, $i \in \mathcal{N}$. The optimal solution $l_i^{t*}(p_t)$ always exists and is:

$$l_i^{t*}(p_t) = \begin{cases} 0 \quad \text{or} \quad l_{i,\max}^t, & \text{if} \quad f_i^{t''}(.) \geq 0, \\ \min\{l_{i,\max}^t, \omega_i^t f_i'^{-1}(p_t/c_i^t)\} & \text{if} \quad f_i^{t''}(.) < 0 \end{cases} \tag{14}$$

where $f_i^{t'^{-1}}(.)$ is the inverse function of first order derivative of the satisfaction function $f_i^t(.)$. For satisfaction function is concave in property II, the optimal solution is $\min\{l_{i,\max}^t, \omega_i^t f_i'^{-1}(p_t/c_i^t)\}$.

The second-order derivative of $\sum_{i \in \mathcal{N}} U_i^t$ is:

$$\frac{\partial^2 U_i^t}{\partial l_i^t \partial l_k^t} = \begin{cases} \frac{c_i}{\omega_i^t} f_i^{t''}(\frac{l_i^t}{\omega_i^t}) & \text{when} \quad k = i, \\ 0 & \text{when} \quad k \neq i, \end{cases} \tag{15}$$

Since diagonal elements of the Hessian matrix are all negative, and off-diagonal elements are all zero. The Hessian matrix is positive definite, meaning strong duality holds, and each individual customer and utility company can simple solve their own local optimization problem determined by (14). Meanwhile, the total power transmission raet $L_t(p_t) = \sum_{i \in \mathcal{N}} l_i^t$ of all customers in time slot t cannot exceed the power transmission rate capacity of utility company in electric power distribution, and is:

$$Q_t^{\min} \leq L_t^*(p_t) = \sum_{i \in \mathcal{N}} l_i^{t*}(p_t) \leq Q_t^{\max}, \forall t \in \mathcal{T}. \tag{16}$$

B. Optimal Responsive Price of Utility Company

In the above subsection, optimal response from customers $l_i^{t*}(p)$ and total power transmission rate $L_t^*(p)$ is obtained. Next, optimal responsive pricing strategy of the first stage will be analysed based on the responses from customers. Knowing the responses from customers, utility company calculates the optimal price by solving problem (12). Plugging (14) into (12), we obtain utility function of utility company corresponding to price vector $\boldsymbol{p}$ as follows:

$$\begin{aligned} &S(\boldsymbol{p}) = \\ &\sum_{i \in \mathcal{N}} \left(w_u \sum_{t \in \mathcal{T}} (p_t - m_t) l_i^{t*}(p_t) - w_c (l_i^{t*}(p_t) - \bar{l}_i^*(p_t))^2 \right). \end{aligned} \tag{17}$$

Given (14), the optimal response of customers as a function of electricity price p_t, we can rewrite the constraints on customers loads as constraints on the prices p_t. From (14) we obtain

$$p_t = l_i^{t*^{-1}}(l_i^{t*}) = c_i f_i'(\frac{l_i^{t*}}{\omega_i^t}), \forall i \in \mathcal{N}, t \in \mathcal{T}. \tag{18}$$

Since (18) is decreasing with l_i^{t*}, the constraints on prices can be writen as

$$p_t^{\min} \leq p_t \leq p_t^{\max}, \forall t \in \mathcal{T}, \tag{19}$$

where $p_t^{\min} = \max\{m_t, c_i^t f_i'(\frac{l_{i,\max}^{t*}}{\omega_i^t})\}$ and $p_t^{\max} = c_i^t f_i'(\frac{l_{i,\min}^{t*}}{\omega_i^t})$. The optimization of $s(\boldsymbol{p})$ in (17) with respect to the price $\boldsymbol{p}$ now becomes

$$\begin{aligned} \max_{\{\boldsymbol{p}\}} \quad & S(p_t) \\ \text{subject to} \quad & p_t^{\min} \leq p_t \leq p_t^{\max}. \end{aligned} \tag{20}$$

The constraints of this optimization problem are linear. To ensure that the solution is the optimum, we need to check the negative-definiteness of the Hessian matrix. In this problem, the negative-definiteness of the Hessian matrix of p is parameter dependent, and conditions have derived in [15].

Theorem : There exists an equilibrium $\boldsymbol{p}^*$ and $\{l_i^{t*}, \forall i, t\}$. Moreover, $p_t^* = c_i f_i'(\frac{l_i^{t*}}{\omega_i^t}) \geq 0$ $(\forall i \in \mathcal{N}, t \in \mathcal{T})$ and $l_i^{t*} = \min\{l_{i,\max}^t, \omega_i^t f_i'^{-1}(p_t^*/c_i)\}$ $(\forall i \in \mathcal{N}, t \in \mathcal{T})$.

Proof: We write the utility function of the whole system as:

$$\begin{aligned} \max_{\substack{(l_i^t, x_{i,a}^t, p_t) \in \mathcal{X}, \\ Q_t^{\min} \leq \sum_{i \in \mathcal{N}} l_i^t \leq Q_t^{\max} \\ l_{i,\min}^t \leq l_i^t \leq l_{i,\max}^t}} \quad & \sum_{t \in \mathcal{T}} U_i^t(l_i^t, x_{i,a}^t, p_t) + S(p_t) \\ \text{subject to} \quad & (1), (2), (3), (19). \end{aligned} \tag{21}$$

The feasible set $\mathcal{X}$ is convex set defined by constraints. Clearly, the optimal solution $(p_t^*, l_i^{t*}, \forall i \in \mathcal{N}, t \in \mathcal{T})$ exists. Moreover, there exist Lagrange multipliers $p_i^{t*}, \forall i, t$, such that (taking derivative with respect to l_i^t)

$$p_i^{t*} = U_i^{t'}(l_i^t, x_{i,a}^t, p_t) \geq 0. \tag{22}$$

Since the right-hand side is independent of user i, the Lagrange multipliers $p_i^{t*} \geq 0$ for all user i. Check the KKT condition for system utility function, since both the utility problem of users and distribution operator and users are concave, the KKT conditions are necessary and sufficient for optimality.

C. Distributed Pricing Incentive DR Algorithm Design

The above theorem derives the existence and uniqueness of a equilibrium in the pricing incentive DR problem, that utility company coordinates customers to jointly compute an equilibrium based on utility functions. Within one loop, utility

Algorithm 1 Utility Company Algorithm

Input: w_u, w_c, m_t, $x_{i,a}^{t*}$, l_i^{t*}.
Output: p_t^*.

Initialize: $L_t(p_t), p_t$ randomly.
while receive information from customers, $t \in \mathcal{T}$ **do**
  Update power rate capacity $L_t(p_t)$ by (16).
  Update constrant boundary $p_t^{\min}$, $p_t^{\max}$ by (19).
  Construct utility funciton optimization problem (17).
  Caculate optimal value of price p_t^* by solving (20).
  Broadcast new value of price p_t^*.
  Receive customers' response of l_i^{t*}.
End

Algorithm 2 Customer $i \in \mathcal{N}$ Algorithm Excuted by EMC

Input: c_i^t, β_i^t, B_{α_i}, B_{γ_i}, δ_i, p_t^*.
Output: $x_{i,a}^{t*}$, l_i^{t*}.

Initialize: $x_{i,a}^t$, l_i^t randomly.
while receive information from utility company **do**
  Update optimal responsive price p_t^*.
  Update power rate constraints $x_{i,a}^t$ by (1)(2)(3).
  Construct customer i utility function $U_i^t(l_i^t, x_{i,a}^t, p_t)$.
  Caculate optimal response $l_i^{t*}(p_t)$ by (14).
  Send optimal response $l_i^{t*}(p_t)$ to utility company.
  Receive electricity price p_t^* from utility company.
End

company updates power rate capacity $L_t^*(p_t)$ according to (16) and further calculates $p_t^{\min}$ and $p_t^{\max}$ at the beginning of time slot $t \in \mathcal{T}$. And then, construct utility function optimization problem according to (17). Eventually, utility company figures out the optimal price by solving (20) and broadcast new value of price p_t^*, and hold on receiving customers' response of l_i^{t*}, as described in **Algorithm 1**. On the other hand, customers hold on receiving pricing incentive information from utility company all the time. At the beginning of time slot $t \in \mathcal{T}$, EMCs update the operational properties of appliances models. By solving the constructed utility function $U_i^t(l_i^t, x_{i,a}^t, p_t)$ locally, EMC in customer i gets the optimal power rate $l_i^{t*}(p_t)$ and $x_{i,a}^{t*}(p_t)$ corresponding to the given p_t^*. And then, send the power rate information to utility company, as described in **Algorithm 2**. Until it is conveged, EMC applies the power rate schedulings to appliances.

The complexity of the algorithm is mainly driven by the number of iterations and the number of users participated in the framework, thus $\mathcal{O}(kN)$ where k is the number of iterations to satisfy the convergence to optimal values with some precision. Indeed, the value of k depends on various parameters such as the number of users and the shape of supply and demand.

IV. Numerical Results

In order to illustrate the outcome of the proposed stackelberg game incentive approach, without loss of generality, we simulate the scenario of one utility company, three type customers with different price sensitivity parameters β_i^t (e.g., $\beta_1 \in (0.1, 0.3]$, $\beta_2 \in (0.3, 0.6]$, $\beta_3 \in (0.6, 0.9]$ denote industrial-type customers, commercial-type customers and residential-type customers, respectively) [16]. Each customer chooses their preference satisfaction parameter β_i^t from customers type category range according to real time energy usage and sensitivity to price in time slot t. Each customer equips at least one kind of appliances with strict energy consumption scheduling constraints corresponding to elastic or inelastic, memory-based or memoryless-based properties. Distribution company chooses balance criteria $\mu = \frac{w_c}{w_u}$ to tradeoff weighted budget of revenue and power provide deviation. Two pricing schemes (i.e., flat-rate pricing and real-time pricing scheme) are adopted in simulation for comparison purpose. In the electric power distribution model, we set an optimization period as 24 hours (i.e., $T = 24$) and the length of each time interval is 1 hour.

As comparing factor, flat-rate pricing has the advantage of simpliness and predictability. Dynamic pricing scheme coordinates customers to participate in load scheduling procedures much more incentively. Distribution company has the authority to decide total revenue and power provide deviation by adopting different balance criteria μ. We evaluate the performance of the proposed distributed pricing game theoretic methods of different combinations of balance criteria for 10 customers. The result of PAR and daily cost of customers is shown in Table. I. It is obvious to find that there are significant superiorities in PAR shifting and daily cost reduction by adopting real-time pricing incentive scheme. Meanwhile, there exists balance criteria $\mu = 0.9$ that can achieve individual optimality with social optimality, i.e., under such balance criteria, when utility company and customers selfishly optimize their own benefits, they automatically also maximize the social welfare.

TABLE I
The Resulting PAR and Daily Cost For 10 Consumers

Real-time pricing	PAR	Daily Cost (Dollars)
$\mu = 0.3$	1.736	167.56
$\mu = 0.9$	1.8437	130.86
$\mu = 1.8$	1.9249	161.64
Flat-rate pricing	PAR	Daily Cost (Dollars)
$\mu = 0.3$	2.32	130.01
$\mu = 0.9$	2.06	133.18
$\mu = 1.8$	2.49	170.39

In order to study the affect of system size on the performance of the proposed game-theoretic framework, we simulate systems with scalable number of customers. Taking customers' preferences into account, we extended the framework under different incentive schemes. Fig. 2 and Fig. 3 summarizes the characteristic factors of balance criteria versus energy cost with scalable numbers of customers. It shows that the energy cost factor will increase linearly as customers scalability increases; but real-time pricing scheme earns more economic saving than flat-rate pricing scheme under the same conditions. It also indicates that balance criteria $\mu = 0.9$ is an optimal

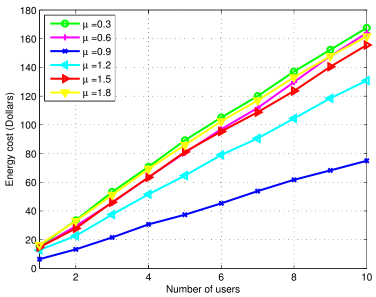

Fig. 2. Real-time pricing vs. balance criteria

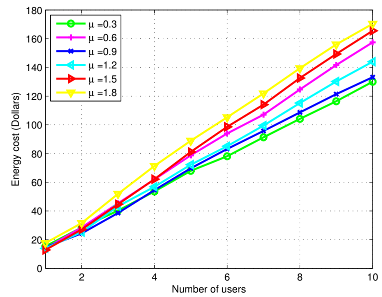

Fig. 3. Flat-rate pricing vs. balance criteria

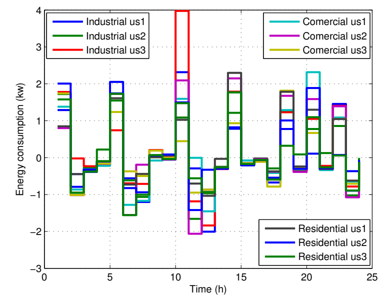

Fig. 4. Scheduling of $\mathcal{A}_3$ for customers

TABLE II
DIFFERENT APPLIANCE WITH MEMORY

	Energy capacity	Maximal power rating
Residential-level	$[3.5, 5.5]$	2.0
Commercial-level	$[4.5, 6.5]$	4.0
Industrial-level	$[5.5, 9.5]$	6.0

tradeoff value that utility company and customers can jointly achieve optimal solutions, as that maximize social welfare.

In order to study the impact of memory-based elastic appliances on the proposed framework for more details, three types of customers choose different scale factors of memory-based elastic appliances as shown in Table. II. Fig. 4 shows the memory-based elastic appliance electricity scheduling under real-time pricing incentive scheme with various energy capacity and maximal power rate. The integration of memory-based elastic appliances including ESSs and EVs provides a certain benefit depending on the pattern or volume of power it consumes. It does not only reduce the peak load but further flattens the demand variation.

V. CONCLUSION

In this paper, we explore the design space of practical and effective schemes for distributed pricing incentive DR for heterogeneous customers in electric power distribution. The interaction between customers and utility company is modeled as a two-stage Stackelberg game theoretic model, where utility company tradeoff weighted budget benefits and PAR, heterogeneous customers optimally schedule appliances power rate based on its preference satisfaction and appliances operational properties. We derive the optimal real-time electricity price and customers' optimal power rate consumption of different appliances. The existence and uniqueness of an equilibrium point has proved. In simulation, we explored the difference between flat-rate pricing incentive schemes and real-time pricing incentive scheme in contributions to overall systems. We find that it is more effective against shifting the shape of demand and greatly maximize aggregate utility of overall system by adopting real-time pricing incentive scheme. In addition, memory-based elastic appliances help reap more benefit and further flatten entire variation. One interesting extension of this work is to consider time dependent pricing design in an oligopoly distribution operator market with competition.

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